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THE RESISTANCE OF THE AIR

DETERMINED

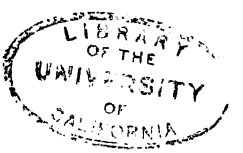
AT SPEEDS BELOW ONE THOUSAND FEET A SECOND

WITH DESCRIPTION OF TWO NEW METHODS OF MEASURING PROJECTILE VELOCITIES INSIDE AND OUTSIDE THE GUN

DISSERTATION FOR THE DEGREE OF DOCTOR OF PHILOSOPHY
IN PHYSICS

BY

ALBERT FRANCIS ZAHM



SUBMITTED TO THE BOARD OF UNIVERSITY STUDIES OF
THE JOHNS HOPKINS UNIVERSITY
MAY, 1898

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L.C.

BIOGRAPHY.

Albert Francis Zahm was born at Lexington, Ohio, January 5th, 1862, and made his collegiate studies at Notre Dame University where he graduated in the Classical Course at the head of the class of 1883. Two years later he took the Master's Degree, and becoming deeply interested in aerial navigation, devoted himself to the study of that subject and its allied branches, especially shop-work, physics, and applied mathematics which later subject he taught in that institution. He took the Degree of Master of Science at Notre Dame University in 1890, the Degree of Mechanical Engineering at Cornell University in 1892, and was shortly after made Secretary of the International Conference on Aerial Navigation of 1893 which he inaugurated and carried to a successful issue with the cooperative of Mr. Octave Chanute. He studied physics at the Johns Hopkins University from February, 1893, to June, 1895, when he was made Associate Professor of Physics in the Catholic University of America, where he has continued to the present date.

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DETERMINATION

OF

THE RESISTANCE OF THE AIR.

WITH DESCRIPTION OF
TWO NEW BALLISTIC CHRONOGRAPHS.

CHAPTER I.

INTRODUCTION.

There has been some discussion as to the law of atmospheric resistance for speeds of 100 to 1000 feet a second; some experimenters maintaining that the resistance increases directly as the speed; others that it increases as the square of the speed; while still others hold that it increases as the cube of the speed, or according to some more complex relation. As a knowledge of the true law is of interest to several branches of science I undertook, at the suggestion of Prof. Rowland, in January, 1895, to devise a method of measuring exactly the resistance of the air to spheres, spheroids and other surfaces of convenient form moving 50 to 500 miles an hour. And here I wish to express my thanks to Prof. Rowland for his able criticisms and suggestions during my preliminary experiments, and to Prof. Ames for some valuable references, one of which set me to working on the electro-chemical method to be described later; also to Dr. Langley, Secretary of the Smithsonian Institution, who very kindly offered me the use of his aerodromic library which contains some rare works of value to this investigation, particularly Duchemin's book on the resistance of fluids.

HISTORICAL SKETCH.

For speeds below one hundred feet a second nearly all investigators agree that the resistance of the air varies directly as the square of the velocity, as first taught by Newton. It is needless, therefore, to notice the experiments made at those speeds. It may

be remarked, too, that it is well known that the resistance of projectiles varies directly as the area of their cross-section and the density of the air.

For speeds above 100 feet a second many resistance measurements have been made. A sufficient account for the present purposes of the earlier of these experiments is given in the following summary by Capt. Bruff of the U. S. Military Academy. §

"The relation between the velocity of a projectile, and the resistance opposed to its motion by the air, has been the subject of experiment from the earliest times to the present day. The most noted experiments on this subject are:

1. "Robins* in 1742 made the first experiments by means of the ballistic pendulum which he invented. His conclusions were, that up to 1100 feet, secs. the resistance is proportional to the square of the velocity; at 1100 ft. secs. the law of the resistance changes; beyond 1100 ft. secs. the resistance is nearly three times as great as if calculated by the law of the lower velocities.

2. "Hutton in 1790 improved the ballistic pendulum and made numerous experiments with large projectiles. His conclusions were that the resistance increases more rapidly than the square of the velocity for low velocities, and for higher velocities it varies nearly as the square.

3. "General Didion† made a series of experiments at Metz in 1839 and 1840 with the ballistic pendulum and spherical projectiles of varying weights. His conclusions were that the law of resistance is expressed by a formula of the general form $R \sim a(v^2 + b\,v^3)$, a and b being constants. This formula held for short ranges, but not for heavy charges and high angles of elevation.

4. "Experiments were, therefore, made again at Metz in 1857, and with electro-ballistic instruments. The conslusions from these experiments were that the resistance varies as the cube of the velocity. Experiments by Prof. Helie at Gavre, in 1860 and 1861 gave practically the same results."

In 1842 Col. Duchemin published his extensive researches,‡ on the laws of the resistance of fluids, in which for air resistance he obtains analytically the formula $R = a\,v^2 + b\,v^3$ for speeds below that of air rushing into a vacuum, or 1366 feet a second, and

§ "Text Book of Ordinance and Gunnery."

* "New Principles of of Gunnery."

† "Lois de la Resistance de l' Air sur les Projectiles."

‡ "Recherches Experimentales sur les Lois de la Resistance des Fluides."

$R=c\ v^3$ for higher speeds, a, b, c, being constants. He compares the resistances computed by these formulae with the measured resistances given by several noted experimenters of the two centuries preçeeding the date of his publication and finds them to agree very satisfactorily.

In 1881 extensive experiments were made at Meppen with Krupp guns and projectiles, the latter being mostly of three calibres length with ogive of two calibers radius. The data of these experiments were discussed by Gen. Mayevski and Col. Hojel of the Dutch Artillery.* Their results are given in English measure by Capt. Jas. M. Ingalls of the U. S. Artillery School as follows: †

Gen. Mayevski obtained.	Col. Hojel obtained.
$V<2300>1370$ f. s., $R\sim v^2$	$V<2300>1640$ f. s., $R\sim v^{1.91}$
$V<1370>1230$ " $R\sim v^3$	$V<1640>1310$ " $R\sim v^{2.23}$
$V<1230>930$ " $R\sim v^5$	$V<1310>1150$ " $R\sim v^{3.83}$
$V<970>790$ " $R\sim v^3$	$V<1150>980$ " $R\sim v^5$
$V<790>0$ " $R\sim v^2$	$V<980>460$ " $R\sim v^{2.5}$

Perhaps the most accurate experiments on projectile resistances were made by the Rev. Francis Bashforth,‡ Prof. of Applied Mathematics in the Royal Artillery Academy of Woolwich. These experiments were conducted at various times from 1865 to 1880. In 1890 Prof. Bashforth, in the revised account of his experiments, stated his formulae of atmospheric resistance as follows:§

For Spherical Projectiles.	For Ogival-headed Projectiles.
$V>1300$ f. s., $R\sim v^2$	$V>1300$ f. s., $R\sim v^2$
$V<1300>1100$ " $R\sim v^3$	$V<1300>1100$ " $R\sim v^3$
$V<1100>1000$ " $R\sim v^4$	$V<1100>1000$ " $R\sim v^6$
$V<1000>840$ " $R\sim v^3$	$V<1000>820$ " $R\sim v^3$
$V<840$ " $R\sim v^2$	$V<820$ " $R\sim v^2$

The discrepancies in the velocity-resistance formulae of these various writers are due partly to the errors of velocity measurement, partly to the methods of interpreting them. The methods of computation need not engage us here as they have not been used in this research; they are fully given in the journals and books referred to above. The velocity measurements, excepting the early, primitive ones, were all made in a similar manner, that is by interposing electric wire screens in the path of the projectile and recording the instant of the wire rupture at each screen by an electric chronograph.

* "Revue d' Artillerie, April 1883, June 1884."
† "Exterior Ballistics."
‡ "Motion of Projectiles."
§ "The Bashforth Chronograph."

A great variety of such chronographs have been used by the various students of ballistics. A good account of these is given by Capt. Ingalls.* Perhaps the most accurate one of recent invention is the polarizing-photo-chronograph of Crehore and Squier† first used at the U. S. Artillery School at Fortress Monroe. Its time-error has not been adequately determined as far as shown in the published account of it.

After a careful study of the various electric chronographs it was thought best to use the purely photographic one, to be described presently, because it is more accurate and offers no screen resistance. When used roughly it will measure time accurately to one-hundred thousandth of a second; when used with due care it measures accurately to about one millionth of a second.‡ The best of the preceeding chronographs, excepting that of Chehore and Squier, measured accurately to about one ten thousandth of a second. This error is so great as to make such an instrument useless for the present research, in which a time error greater than two millionths of a second is inadmissable if the retardation is to be measured accurately to one per cent.

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* "Ballistic Machines"

† "The New Polarizing-Photo-Chronograph."

‡ Greenhill, Philosophical Magazine, June, 1890.

CHAPTER II.

PLANS. SUGGESTIONS. PRELIMINARY EXPERIMENTS.

There are two general methods of measuring the pressure of impact between the air and any body in relative motion with it; (1) that in which the air is driven against the body; (2) that in which the body is driven against the air. The first method has been frequently used for speeds below 100 miles an hour, while for higher speeds the second method alone has been employed, though doubtless both might serve for all velocities from zero to one thousand feet a second for bodies of not too great dimensions.

First General Method.—The method of driving the air against the body offers the inviting advantage that the resistance can be measured directly and with perfect nicety, as for example, by directing the draft downward and suspending the body in it from the arm of a sheltered balance. But the difficulty of measuring the velocity of the draft is quite formidable. It cannot be done accurately by any of the standard anemometers; for they do not give consistant readings even for common wind currents. It might be conveniently measured by some form of pressure guage, for example a small sphere hung in the draft, or a sharp-mouthed tube opening into it and communicating the pressure to a manometer; but such instruments have yet to be graduated for high speed currents. The speed might also be found by collecting the stream of air in a large, well poised tank, and observing the volume from instant to instant, though this would be costly unless one could find a tank available for such use.

A second objection to driving the air against the body is the non-uniformity of speed at all points of a cross-section of the stream of air, the sides moving more slowly than the middle of the stream, owing to the friction against the wall of the pipe or funnel; but doubtless uniformity could be secured by some device for damping the central parts of the stream.

A third objection is the cost of producing a current of several hundred feet a second. The quantity of air set in motion per second increases directly as the velocity, and the energy of each pound of it as the square of the velocity; hence the power required to maintain a stream of given section is proportional to the cube of the cur-

rent specd. Yet if the other difficulties could be overcome this one would not render the method impracticable. The current is needed for but a moment while the pressure against the body is being observed; hence if one had access to a large compression plant it would be sufficient to take air from the mains for a minute now and then. There are some compression plants—such, for instance, as are used for Bessemer converters—where engines of some hundreds of horse-power supply large streams of air at a pressure of 20 to 30 pounds above that of the atmosphere. Such a plant would furnish a stream moving many hundreds of feet a second though of not very large cross-section. Hence it would seem that the resistance of small bodies might, by this method, be measured for a great range of velocities without much expense for apparatus.

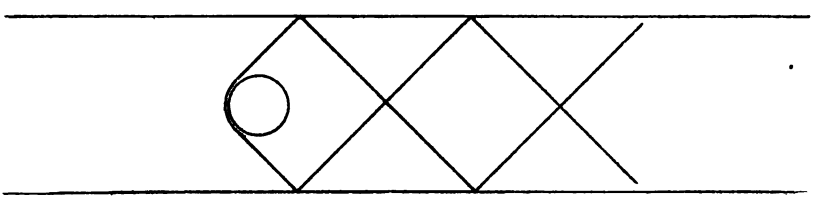

Fig. 1.—Photograph of a Bullet in Flight with Air Waves about it.

It would be interesting to calculate what must be the least diameter of a pipe to make the current pressure against a given body placed at its center the same as for a pipe of infinite diameter. At first thought it would seem that a four inch ball in a stream of air having the velocity of sound, would meet a pressure hardly greater in a twelve inch pipe than in one of infinite diameter; for any given sectional slice of air would pass the ball while the compressional wave was running out from the ball to the pipe-wall. Judging by the photographs of bullets in flight, the appearance of the waves would be something as shown in Fig. 1. This figure manifests a short air prow before the ball, and shows the waves running laterly outward from the ball and reflected from the walls of the pipe. The illustration is from a conjectual extension of Boy's photograph of a bullet in flight. (See Nature, Jan. 1893; also Nichol's Outlines of Physics, article 303.) If this conjecture be correct, the blast required for balls of moderate dimensions might easily be obtained. I have, however, not tried to develope this plan because the second general method promised to be less costly and more convenient.

Second General Method. — Something may be said for and against the second general method, or that of driving the body

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against the air. It is much cheaper, at high speeds, to propel a ball against the air than to propel the air against the ball. The velocity of the projectile can be very exactly measured, and there is no pipe-wall to be considered. The density and the temperature of the air can be exactly observed. But the great difficulty is to find the pressure between the ball and the air; for to do this we have not only to measure the velocity of the ball, but also to determine accurately its change of velocity in a small fraction of a second, from which the pressure can be calculated,—unless the ball is carried on a car or something bearing a pressure guage.

There is a prospect that we may soon be able to measure directly the atmospheric resistance up to 200 feet a second, or more, by the second general method. Experiments with rapid elevated cars, notably those recently made in Belgium, give promise of practically realizing speeds up to 150 miles an hour, or 220 feet a second. With such a car at one's command, it would be merely a matter of detail to measure the resistance of almost any shaped body with great accuracy. Another plan, for small bodies and speeds up to a few hundred feet a second, would be to mount the object on a long whirling arm whose axis moved rapidly forward so as always to encounter new air. But in this research the most practicable way was to hurl the body freely through the air and find the resistance from the observed retardation—a plan, however, which made a wide range of speeds possible.

Having adopted this method it was apparent, that to make it most accurate, the projectile should be the lightest that can be made without risk of rupture or deformation; it should move through homogenous still air; it should encounter no resistance from the instrument used to measure its velocity.

The Propelling Mechanism.

Many plans suggested themselves for driving the projectile. The body could be allowed to drop from successive elevations and the time of fall observed for the whole height of each elevation or at three points along the height as might prove most convenient for measuring the velocity. To obtain great speeds an officer of the Weather Bureau suggested that the ball be dropped from a cliff or captive balloon, which latter, he thought, could be borrowed for the purpose from the War Department. Prof. Rowland advised, for some speeds, the use of a lofty sheltered tower, an old-fashioned catapult or an air-gun, powder being objectionable indoors because of the smoke and smell, also because it might injure a wooden ball

by singeing and daubing. There is a deserted shot-tower in South Baltimore which would give a free fall of 350 feet, and the Washington Monument at Washington would give a free fall of 500 feet. The utmost attainable speed from the latter would be 160 feet a second, and the Monument would be available only before nine A. M. and after four P. M. I did not think I could design a cheap catapult, and perhaps not even an expensive one, that would give a great velocity. So, for the higher speeds, nothing seemed available but some kind of gun, though, of course, for low velocities a tower would serve very well and may be used in extending this research.

I then designed an air-gun and a steam-gun. To obviate the need of an expensive air-compressor, water from a boiler feed pump was to be forced into the bottom of a large pipe compressing air above it as it rose to the top of the pipe. A large rotating valve suddenly turned by the torsional force of a strong steel rod, was to admit the air to the gun-barrel, which was simply a seamless drawn brass tube, ten feet long. The steam-gun was of similar design, but having the barrel surrounded by a steam-jacket to prevent the ball being wetted by moist steam. I calculated that an initial pressure of 100 pounds to the square inch would easily propel a wooden projectile such as I expected to use, 500 feet a second. Hence both these guns seemed practicable; but I am now glad that they were never tried because the powder-gun is so much simpler.

On the 17th of August, 1895, through the courtesy of Prof. Alger of the U. S. Navy, I was taken to Indian Head to witness some gun tests, and was there offered an opportunity to make a rough estimate of the charge of powder required in my research. A polished pine ball, 2.24 inches in diameter, was fired from a 6 pound Hotchkiss rifle, and the velocity roughly measured. The experiment showed that 11 grams of black powder would give a ball weighing 102 grams a speed of 400 feet a second. This encouraged me to procure a gun of my own and try smokeless powder.

A few days later I attached a small steel explosion chamber to the end of a brass pipe 2½ inches in diameter and 7 feet long, and placed a polished pine ball in the breech five or six inches ahead of the powder to save bruising the wood. Dupont's white smokeless powder was used, back of which was just enough black powder to ignite it, and back of that a gun cap which I struck with a hammer. The ball was caught in a barrel of paper twenty feet distant, and when examined showed a black belt of smoke ⅛ of an inch wide extending clear around it, which indicated that the ball did not rotate while in the gun, though it fitted neatly. As this fact is im-

portant I fired the bullet again, using only black powder. All the back hemisphere was blackened, and the gun was blacked to the muzzle; but the front of the ball remained clean; hence it could not have rotated within the gun.

This fact was very gratifying; but another appeared which foreshadowed much annoyance and difficulty. The gun would not shoot straight. The ball veered right and left as well as up and down. Some such difficulty had been anticipated for elongated projectiles, but not for polished spheres at such short range. Prof. Rowland had advised the use of a stretched wire to guide the long bodies; and it now appeared that the same would be needed for spheres also.

When I again resumed the work in the summer of 1896, a steel piano wire ⅛ inch thick, was led from a diametral cross piece in the breech of the gun straight through to the other side of a forty foot room, through a horizontal barrel of bran which served as a target, fastened to the brick wall and stoutly stretched by a turn-buckle. The pine ball had a sleeve threaded through it to make it glide nicely along the wire, but struck the open barrel-head on its *edge* instead of its center, crushing the staves and breaking the ball and wire. This result ought to have been foreseen; for whatever vibration was started in the wire at the gun's muzzle would be intensified as the ball approached the other fixed end, just as a limber saw whips more and more violently as it is drawn through its cut. I then saw that to carry out Prof. Rowland's suggestion, the wire should be massive to make it vibrate less readily and stretched plumb, to prevent sagging; but as I had not a tower to experiment in, I gave up the use of a guide wire and used a larger gun, hoping thus to secure a more even trajectory.

A solid ash bullet was next shot from a four inch gun to ascertain why I had not been able to hit the head of a barrel thirty feet away. The ball was turned accurately to 1-100 of an inch in an engine-lathe, polished, and balanced so that it would float in any position in a basin of water. Still I could not shoot into the end of a twenty inch barrel with certainty. Hence I concluded that, as the ball was a perfect sphere, flying without rotation, the head resistance of the air could not deflect it and the side veering must be due to the uneven pressure of the gases at the muzzle of the gun.

Finally, I decided to rigidly guide the bullet till the gases should die behind it. A wooden rail-way with three rails was fixed before the gun and along it were placed screens through which the ball must pass. This device proved very successful. The gases could not reach the farther screens and the ball shot much more nearly

straight to the mark. The detailed description will be given in the sequel.

Careful attention was next given to making the bullets not only truly spherical but also as light as may be without risk of deformation during flight through still air. Accurate sphericity was easily attained by turning in an engine-lathe with firm slide-rest; but the attainment of the utmost lightness proved more difficult. Solid pine balls were tried but they were too soft to be used more than a very few times. Oak balls were hard enough to be used many times when caught in a barrel of cotton-waste, but they were excessively heavy. Ash balls were lighter and hard enough to be fired a score of times without bruising or roughing them seriously. So these were used for practice and for the first determination of the curve of resistance.

Still lighter bullets were sought for and many plans considered for making them. Hollow ash spheres of half the weight of solid ones were used for a while and proved fairly satisfactory, but they were too short-lived and expensive to be used for many observations, since it takes a mechanic nearly three hours to make one properly. Besides, wood is objectionable for long service because of uneven shrinkage. Cheap, frail balls were thought of to be used only once. They might be of glass, *papier mache* or possibly plaster of Paris. Hollow, hard rubber balls of fine exterior surface were offered by the Butler Hard Rubber Co., of New York, but they were not of very uniform diameter and thickness of shell.

Finally, I tried to obtain hollow metal spheres that should be cheaper, lighter, and stronger than the hollow ash ones. Such a bullet was made from sheet brass 1-50 of an inch thick, by spinning hemispheres and soldering them together. It shot across the room at high speed without breaking but, being crude and uneven, it flew aside, missed the target and was destroyed. After considerable correspondence, I could find no one who would undertake to spin and solder such spheres if they had to be accurate to 1-100 of an inch, though the task seems easy. So a process was devised for forming, by electrical deposition, very thin, closed metal shells of uniform thickness, and accurate, smooth, spherical surface, to be filled with compressed gas. In the meantime, however, I had proposed to the American Machinist the problem of making such shells of steel; and a great variety of suggestions were obtained; among others, a method for forming the shell from a single sheet. Many manufacturers offered to make dies for pressing true hemispheres, the price varying from $25 to $150. After a series of private references from

one manufacturer to another, I at last found a firm that had such dies and were making balls of the right diameter from sheet steel, 1-75 of an inch thick. The Ferrachute Machine Co. also offered to make a die for $25, and to form the shells from 1-100 inch sheet steel, guaranteeing them not to vary more than 1-100 of an inch from true sphericity. These would weigh about ¼ as much as solid ash balls and enable me to easily measure the retardation accurately to one per cent. But when I asked them to make thinner shells, say from 1-1000 inch steel, they promptly demurred, fearing the sheet would buckle in the die during the process of pressing.

The Chronograph.

Quite a number of plans suggested themselves, also, for measuring the projectile velocity. There was the known method of stretching three electric wires in the path of the bullet and recording the rupture of each wire by the marking of an electro-magnet on a rotating drum or falling bar of zinc. But Prof. Rowland very wisely cautioned me against the use of an electro-magnet in any instrument which was to give accurately the same effect twice in succession. Boys' photographs of small bullets in flight suggested another method. These photographs show the aerial wave crests running away to right and left of the bullet's path so that each crest is a right line making a definite angle (fig. 1.) with the path of the bullet. The tangent of this angle is equal to the speed of the wave divided by the speed of the bullet; hence if the velocity of the wave was known that of the bullet could be computed, and vice-versa.

The two most promising electrical methods considered were that of Crehore and Squier already mentioned, and an electro-chemical method which I devised in February, 1895. Both methods, unfortunately, require three wires to be broken by the bullet. This might not be objectionable in ordinary ballistic work where one has merely to determine the velocity of massive metal projectiles; but in this research wire screens could not well be admitted, because of the resistance they would introduce, and because of ther destructive effect on wooden balls or very thin shells which I wished to employ; also because a wire is not instantly severed by a bullet either mechanically or electrically, owing to stretching and inertia which prolong the contact. Still, the simplicity of this method was so inviting, that I temporarily abandoned a good photographic one which I had been contemplating from the beginning of the research and some years previously, in order to test an electro-chemical method which may be worth a passing notice.

In February of 1895 the Compte Rendus described a new method of recording the period of an alternating current. In principle it was an application of the old Bain printing telegraph. A paper soaked in a solution of ammonium-nitrate and potassium-ferrocyanide, was fastened to a chronograph drum, and against it was pressed a sharp steel electrode, the other electrode touching the drum shaft. A blue mark was made when the current flowed from the stylus into the paper, but no mark when the current flowed reversely.

This seemed applicable to the research in hand; for if a very rapidly alternating current were used in series with the three wires to be cut by the bullet, the time between consecutive cuttings would be shown by the number of alternations between the breaks in the drum record. To standardize this a second stylus in a continuous circuit with a standard tuning fork, or other chronometer, might write its record beside the first one. Or if the interruptions of the record could be made sharp enough, a continuous current might be passed through the three wire screens and the time-interval between the consecutive cuttings determined from the linear distance between the breaks in the record.

To test the possible speed of electro-chemical tracing and the sharpness of interruption, I slipped the lid of a tin can as a cap over the end of a motor pulley, pressed against it a disk of paper moistened with the Bain solution, and set the motor running at 1600 rotations a minute. Drawing the stylus from the center of the disk to the circumference I obtained a bright, blue spiral all the way from the center of the disk where the speed was zero, to its extreme edge which was moving 35 feet a second. The voltage was 100, the current 1-10 ampere; and it seemed that, with a sufficiently high voltage, there ought to be no limit to the tracing speed.

I then cut the circuit some yards from the motor, fastened one section to a brass plate lying on a bench and, while an assistant tapped the other section sharply on the plate, I drew the stylus across a fresh paper disk a sketch of which is herewith presented (Fig. 2). The "makes" in the record are quite sharp, considering the crudeness of everything; but the "breaks" are drawn out, owing, doubtless, to the slow severing of the wire and plate.

The use of paper is objectionable for records that are to be exactly measured; hence something was sought which would absorb the solution without stretching, and dry without shrinking. I first tried a clay pipe. Having marked two points on its handle 6 inches apart with sharp dividers, I soaked it in the solution and applied the dividers again, finding the distance exactly the same. I then

tried a clay disk; also a clay drum made of a filter-tube fitted on the motor shaft and turned up with a pen-knife until it was smooth and true. The records on these were beautiful; for the clay was pure white and the spirals clear blue.

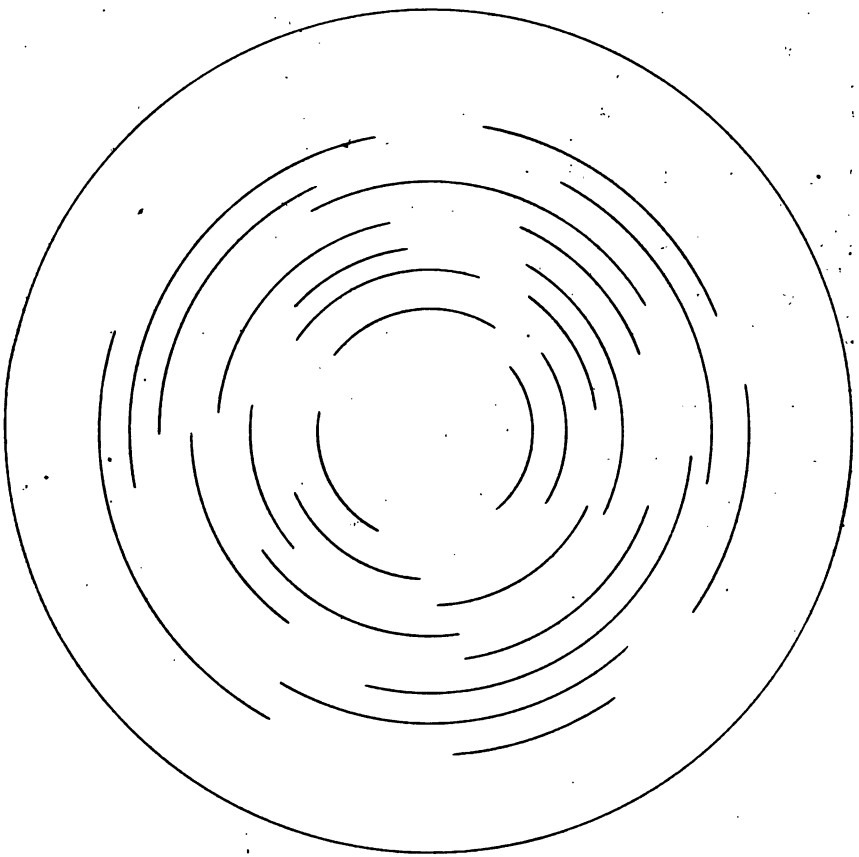

Fig. 2.—Sketch of Electro-Chemical Record.

It now seemed certain that the principle of the electro-chemical method was a good one, and many details suggested themselves. The bullet could be made to declare its position either by making or breaking the circuit at each of the three, or more, stages of its path. If a grating of very fine wires, alternately positive and negative, were placed at each stage, the bullet might for an instant short-circuit those struck, thus giving a sharp "make" to the blue record which would cease before the bullet reached the second stage. Or, if found better to have the ball break the current at each stage, any

of the methods practised in gunnery for automatically closing the circuit between consecutive ruptures could be used.

Finally, to obviate the need for a chronometer to standardize the record, I proposed to have a long, true clay drum, or plate, mounted on a massive metal holder and droppod freely down a vertical guide, while the stylus traced a single rectilinear record containing the three "makes", or "breaks", which should time the transit of the projectile. Instead of soft clay it might be well to use hard clay, like that of a porous battery jar, ground and polished to a perfect surface which would keep the stylus cleaner than a soft surface and not abrade it so readily. A large number of records could be placed on a single plate or drum, each record being but a single fine straight line; and, if desirable, the records could be dissolved away (by K O H for example) and the same piece of clay used over again. This, no doubt, would be a very convenient and satisfactory method for measuring projectile velocities, though for measuring resistances the light-beam method is more accurate.

It would have been interesting to determine the possible accuracy of the electro-chemical chronograph. If a circuit were used having a little higher voltage and less self-induction, the tracing might be made 1000 inches a second, and the break located accurately to 1-1000 of an inch, thus measuring time accurately to 1-1,000,000 of a second. But I did not push the investigation further because the photographic method which I had been studying all along, seemed best adapted to determine the resistance of light and frail projectiles.

Photographic Method.—The idea of using a falling photographic plate in a tall closed box as a chronometer, first occurred to me while at Cornell University in 1889; but the thought of using the chronometer for measuring the velocity of projectiles came after a suggestion of Mr Uhler, assistant in the Johns Hopkin's Laboratory. Mr. Uhler advised me to throw three powerful streams of light across the bullet's path, condensing them by lenses to a focus where they were to be cut, and then passing them into three drum chronographs carrying sensitive films. I did not then see how to compare the records taken on three different chronograph drums, unless they were all mounted on one large, rigid shaft, though it occurred to me later that they could be compared by making a sharp electric spark at the same instant before each drum and measuring on each the time between its spark and bullet records. So, instead of following Mr. Uhler's good suggestion, I decided to lead one continuous light-beam back and forth three times across the bullet's path, then

through a condensing lens to a focus on one sensitized drum. The shadow of a tuning-fork prong partly in the beam would serve to standardize the speed of the drum. Prof. Rowland suggested replacing the fork-prong by a serrated disk having 100 teeth and making 100 rotations a second, which he said could be done satisfactorily with one of his small motors. The period between the interruptions would thus be 1/10,000 of a second. But as I was to leave the University before the research could be finished, and afterwards might not have the convenience of current and motor, I did not venture to carry out Prof. Rowland's suggestion. I also abandoned the plan of using a tuning-fork and drum as being rather cumbersome; and so, at last, adopted the photo-gravity method which I had conceived in its essential elements some years previously. It will be seen later that instead of the contemplated zigzag beam, three straight sunbeams were thrown across the bullet's path and diverted to the same camera. This was to allow the beams to be very thin and instantaneously extinguishable, while at the same time intense enough to produce strong records.

In order to test how sharply a light-beam record could be interrupted, I had a carpenter to make a columnar box, five feet high, three by five inches inside section, and having two grooves down which a board carrying a sensitive plate should fall. Placing a small condensing lens of ⅛ of an inch aperture in the side of the box, I allowed sun-light from a vertical slit, measuring ⅛ by 1/100 of an inch, and 50 feet distant, to come to an approximate focus on the falling plate, which was dropped from a latch. While the plate was falling I struck the beam with a lath, moving it swiftly to and fro. After developing the plate, the record was examined under a micrometer microscope. This showed that the edge of the interruptions could be located accurately to 1/5000 of an inch, which, perhaps, is nearly as close as one dare measure a film. As the plate was moving 100 inches a second, this test showed that the time of interruption could be fixed accurately to 1/500,000 of a second which was near enough for my research.

By increasing the speed of the plate and using strong light, I think the limit of accuracy can be made less than one millionth of a second, and I hope to reduce it below one tenth of this for another purpose. I propose to move the plate 1000 or 2000 feet a second, and guage its speed by some ordinary method, or by photographing an electric spark whose light goes to the plate over two routes, one being several hundred feet longer than the other. If the two images of the spark, which coincide when the plate is still,

can be separated by 1/50 of an inch by its motion, which seems feasible, the speed of the plate can be determined accurately to one per cent. It can certainly be determained with this accuracy by ordinary methods, and, in either case, the record of so swift a plate will show time intervals of one hundred millionth of a second.

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CHAPTER III.

COMPLETE APPARATUS. RECORDS.

Having finished the essential preliminary experiments at the Johns Hopkins University during the winter and spring of 1895, I continued the research at the Catholic University of America during the summer vacations of 1896 and 1897.

The general arrangement of the assembled parts of the apparatus, constructed at the latter university in the summer of 1896, is shown in fig. 3. The gun is at G, and in front of it are eight screens, S–S, for stopping the blast when the gun is fired. The ball emerging from the screens passes through still air cutting three sunbeams, and strikes the target. The sunbeams come from the mirrors, M M M, pass through the slits, A A A, B B B, and are reflected from the right angle prisms, R R R, to the camera where they are focused on a photographic plate, P. When the plate falls the sunbeams trace on it three fine, straight lines close together, each line being momentarily interrupted when its tracing sunbeam is cut by the bullet. The position of these three interruptions serve to calculate the bullet's velocity and resistance.

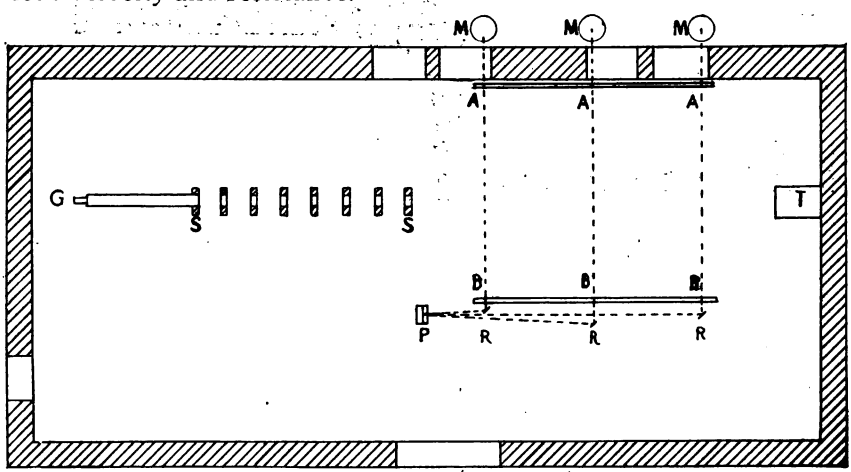

FIG. 3.—Plan of the Complete Apparatus.

The apparatus is located in one of the private laboratories of the Department of Mechanics, a basement room 25 by 50 feet, and ten feet high. The room is free from drafts, except when it is desired

to clear away the powder fumes; the temperature uniform; the windows darkened for convenience in handling the light-beams. The plane of the light-beam, gun and target is midway between the floor and ceiling.

The Gun.

The gun is an ordinary seamless-drawn brass pipe, seven feet long, four inches inside diameter and ⅛ of an inch in wall thickness. Its breech is closed with a common brass pipe-cap, into which is screwed a small, stout toy cannon. This also is of brass, four inches long, one inch inside diameter, and provided with a gun lock and ignition tube at its breech. Thus the whole consists of a stout little cannon screwed into a big frail one. A charge of 25 c. c. Dupont white smokeless powder, primed with two c. c. of black

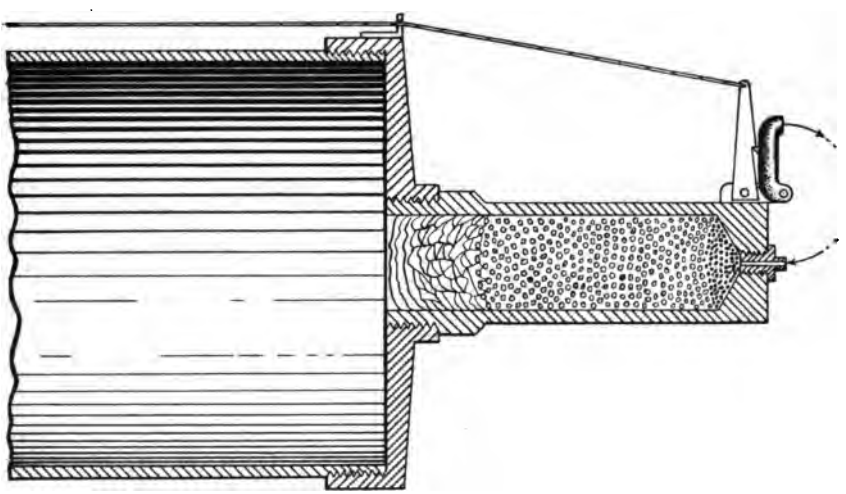

Fig. 4.—The Gun Breech and Explosion Chamber.

powder gives a solid four inch ash ball a speed of 500 feet a second. The gun is fired by a cord tied to the camera trigger to be shown presently.

The Bullets.

Only a small variety of projectiles have thus far been used, and these have been spheres 3 63/64 inches in diameter, this being the size that snugly fits the brass pipe gun which is nominally of four inch bore. For the first records solid ash balls were used and after that, hollow ash balls of one half inch wall thickness. The latter are preferable at safe velocities because they weigh but one half as much as the solid balls; but for velocities above 500 to 600 fee

a second, they are too short lived. At such speeds they usually crack at the first or second shot, and break open at the succeeding ones even though meeting nothing more resistant than the target-paper and the cotton waste back of it which stops their flight. The hollow ones weigh about 150 to 170 grams; the solid ones about 300 to 360 grams. They are turned truly spherical and polished, but not balanced, as they are found seldom to rotate so much as half a turn during flight. This can easily be shown by suspending small leaden shots by threads at intervals along the bullet's course, the shots as a rule all striking the ball on the same hemisphere.

If the gun could be made to shoot straight, smaller balls such as those first experimented with, would show a greater retardation. For, since the retardation varies directly as the surface and inversely as the mass, it must, for a given density, vary inversely as the diameter of the ball. But as the crude wooden screen trestle at present employed will not deliver the projectile along a sufficiently precise and predeterminable trajectory, I have not tried to use smaller balls nor forms of surface other than spherical.

The Screen Trestle.

To promptly check the gases and guide the balls during their subsidence, the device shown in fig. 5 is placed before the gun. It is a heavy, fourteen foot trestle on which are mounted eight screens two feet apart, and having holes just large enough to let the ball

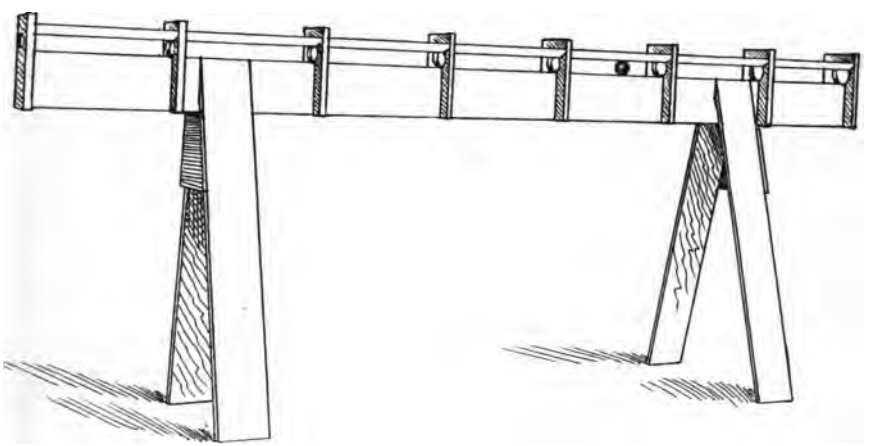

FIG. 5.—The Screen Trestle.

pass through. The screens are rough pieces of plank eight inches wide, one and three-eighth inches thick, and bolted upright one behind the other on the trestle. To direct the ball three ash guide

bars run through the screens, being adjustable in slots so as to just hug the ball neatly and prevent side play. This trestle when in condition works very well, stopping the gases at about the sixth screen and letting the ball emerge into still air; but owing to the warpage, it requires frequent adjusting, and sometimes a sand paper drum has to be run through it to true the guide bars.

The Gun-Brush.

To keep the bullet smooth it is advisable to clean the gun after each shot as nicely as possible. At first a cannon sponge obtained from the U. S. Arsenal was used; but it left dry grains of powder behind, that embedded themselves in the bullet, thus roughing its surface. I then had a cannon brush made of the best Russian bristle arranged in ½ inch rings around a cylindrical block 8 inches long. To this was fastened an iron gas pipe handle 21 feet long which enabled the gunner to sponge both the trestle and gun after each shot.

The Target.

The target is a common lime barrel, filled with cotton waste to catch the ball without injury. Before the barrel is hung a piece of blotting paper, soaked with water about where the bullet is to strike. The ball cuts a sharp hole through the paper from which one can ascertain by a guage stick whether the light beams were cut centrally. This corroborates the story of the chronograph.

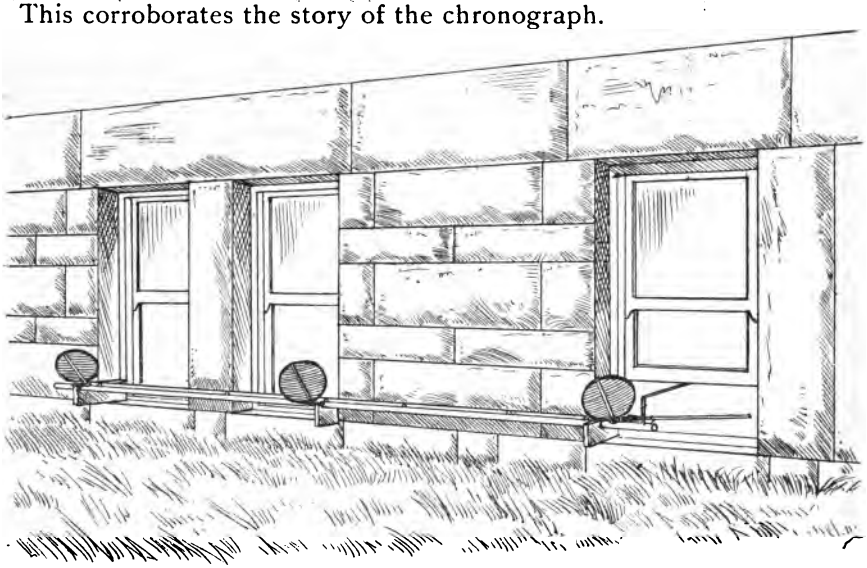

FIG. 6.—The Triple Port-Lumiere.

The Mirrors.

Sun light has been the most available; therefore I devised a triple port-lumiere to throw three parallel beams across the bullet's path. This (Fig. 6) consists of a horizontal 2 inch gas pipe, resting on three wooden bearings, attached to the window sills and having through it, at 7 foot intervals, three shafts carrying plane mirrors. Two handles pass out from the room, one to rotate the pipe the other to rotate a worm shaft mounted on it, which engages the mirrors. By this means the gunner can hold the streams of light on the guage slits while the trigger at the camera is made to drop the plate and fire the charge.

The Guages.

To accurately space the sunbeams two horizontal bars, each carrying three vertical brass slits are used; one bar mounted on the wall by the windows, the other on a trestle (Fig. 7.) at the other side of the bullet's path. The slits are 1/100 of an inch wide and all accurately spaced by comparison with a brass bar having two fine marks on it 7 feet apart; this, unfortunately, being the largest spacing the windows would admit. The light from the mir-

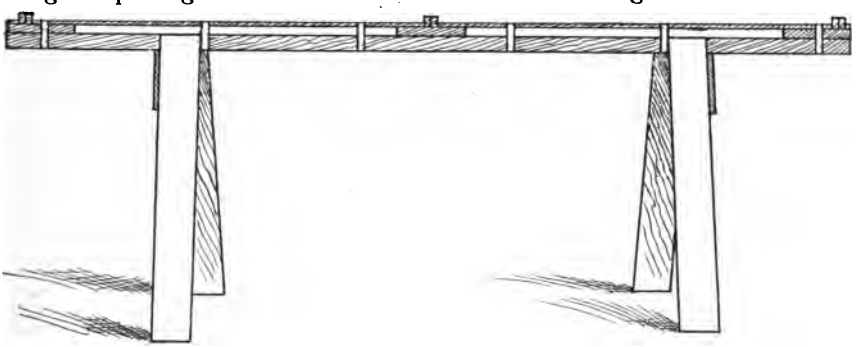

Fig. 7.—The Guage and Reflector Trestle.

rors passes through the wall slits spreading out cone-like till it is all stopped at the trestle opposite, except what can pass through the slits there on its way to the camera. It is thus seen that the *effective streams* are ribbons of light 1/100 of an inch thick, parallel, accurately spaced and square with the gun. In practice the ribbons are ⅛ of an inch wide up and down.

The Reflectors.

The streams of light on passing the guage-slits encounter three reflectors which divert them to the camera. These reflectors are ½

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inch right angle prisms, each adjustable about two axes by screws so that the operator at the camera can quickly turn the beams in any direction. If the reflector trestle were immovable, the light from the reflectors would preserve a fixed direction; but the trestle shifts occasionally from warping. Thus, while the adjusting mechanism would not be needed on a better mounting, it is very convenient with the present one. The streams entering the camera hole cross each other at a very fine angle as shown in Fig. 3, and fall close together on the sensitive plate.

The Zero-Marker.

It was found impossible to open and close the camera shutter by hand quickly enough to let the streams properly mark their zero positions on the plate before it dropped; hence the bottom portions of the record were blurred from over exposure before the plate could get under way. This difficulty was overcome by use of the little instrument shown, half size, in Fig. 8. It is a thin disk, having a radial slit, and mounted on a handle by a spiral spring stiff enough to rotate it very suddenly. The disk is held cocked before the open

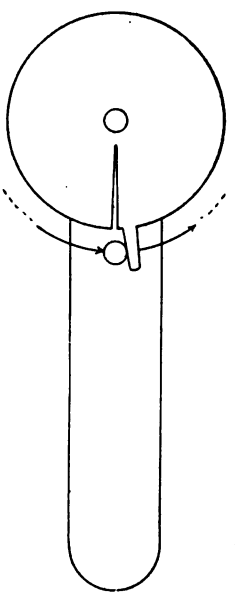

FIG. 8.—The Zero Marker.

camera-hole and allowed to snap. As the radial slit flies past the hole light is admitted much less than 1/1000 of a second, thus mark-

ing the bottoms or zeros of the straight lines which are to be traced when the plate is dropped. The shutter is then closed to be opened automatically when the plate is falling rapidly.

The Camera.

General Description. — The camera is a columnar cast-iron box, 5 feet high, 3 by 5 inches inside section, and provided with two interior side grooves, down which a massive iron bar drops freely, carrying with it a photographic plate, 2 feet high, by 2½ inches wide. At the bottom of the camera is a cushion which yields to the impact of the falling mass so as not to endanger the plate; and in the back of the camera is a windlass which draws the plate and holder up to their initial position where they are held by a firm latch ready for another drop. After each fall of the plate, the camera may be displaced sidewise, say 3/16 of an inch, while the shutter and lens remain stationary; thus it is possible to take a dozen or more records on one plate. Outside and back of the camera is a trigger whose mechanism drops the plate, opens the shutter and fires the gun at pre-adjusted time intervals.

Detailed Description. — A sectional view of the camera with trigger mechanism and supporting timber is given in Fig. 9. The plate, P, clamped to the bar, B, is shown suspended from a latch, L, which slides in a horizontal hollow post in the back of the camera, actuated by the lever, T. called the "trigger." On a step near the bottom of this lever is suspended a drop-weight, W, from which a cord runs to the gun; and above this drop-weight hangs a second one, W′, from which a cord runs to the camera shutter. Thus when the trigger is pressed, the plate and drop-weights fall; and when the latter stretch the slack from their cords, the shutter opens and the gun shoots. By adjusting the slack, therefore, the time intervals are fixed for the shutter and gun. In practice the plate drops about an inch before the shutter is jerked up, about eight inches before the gun-lock is released, and twelve before the ball cuts the first sunbeam. A single drop-weight could work the strings of both shutter and trigger; but two weights give a greater range of adjusrment.

The falling plate and holder strike an ash block tipped, top and bottom, with rubber and resting on a thin steel plate which, in turn, rests on three felt ones. The whole forms a very durable cushion of just the right elasticity, not causing the plate to rebound to the sunbeams. The ash block which fits the side-grooves nicely, can be drawn up by the windlass, as shown, and falls again freely, after leaving the plate suspended on its latch. The lens-holder, v, is shown

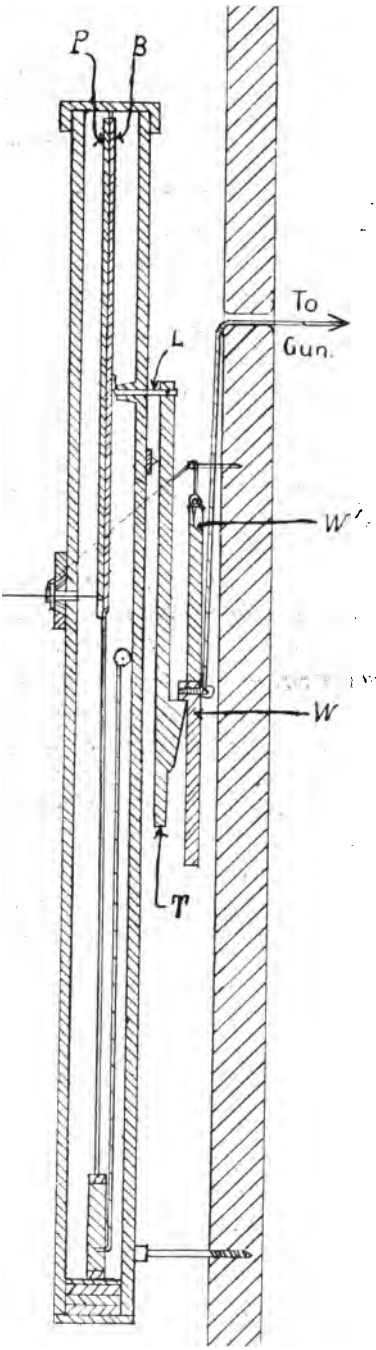

FIG. 9.—Vertical Section of the Camera.

dove-tailed into the camera-front, to allow cross motion in setting for a new record. This is better portrayed in the perspective view, Fig. 10. The camera hangs from a post by a cross bar, w, and is

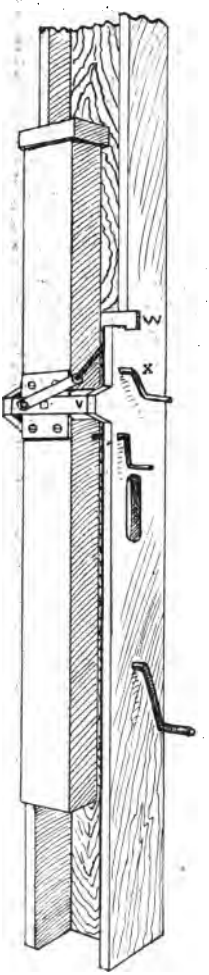

FIG. 10.—The Camera Mounted on its Post.

pushed side-wise by the screw, x, while the lens-holder, v, remains stationary. The bottom crank works a windlass for lifting the camera upon the post when brought from the dark room.

The lens is of 2½ inch focal length, ⅛ inch square aperture, and adjusted so as to throw an image on the plate a little out of focus to prevent flaring. Thus sharp zeros and clear tracings are secured.

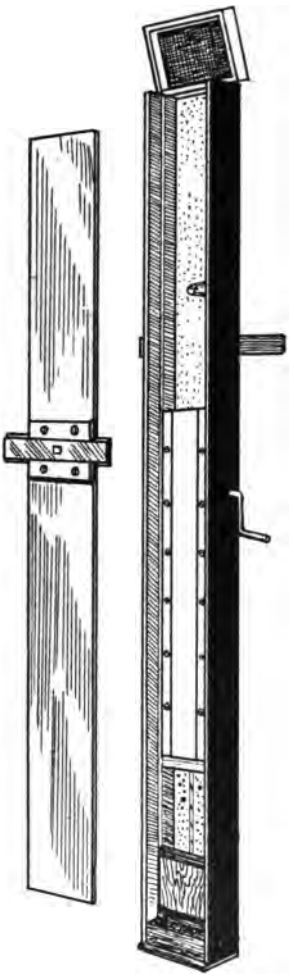

Fig. 11.—The Camera with its Front Removed.

Plates.

The most sensitive Cramer plates, known as the "Cramer Crown," have been used, though any rapid kind would answer. They are cut from regular 20×24 inch sizes and measure, 24×2½×⅛ inches. They are held as shown in Fig. 12, which represents a section of the plate and holder. The body of the holder is an iron bar 25×4×⅜ inches, having a sheet of rubber pasted on one face to serve as a soft backing for the glass plate. Brass strips running the full length of the plate are clamped against it by screws, thus holding it with even pressure. The bottom end of the plate abuts against a brass

piece, also which prevents sliding due to the falling impact. The developing is best done rapidly so as to bring out only the tracings

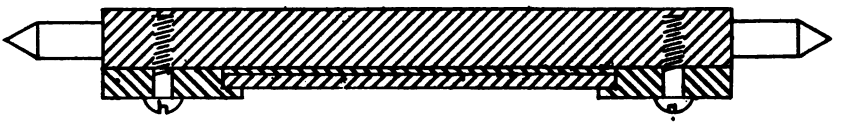

Fig. 12.—Section of the Plate Holder and Plate.

and leave the rest of the plate clear. Any good developer will serve for such simple work.

Observations.

During the progress of an experiment notes were made of the temperature, barometric pressure, charge of powder, mass and condition and flight of ball, etc. The accompanying page (table 1) from the laboratory note book illustrates the nature and variety of these observations.

TABLE I.

Observations of Sept. 3rd., 1897.

Ash ball (7); diameter, 3''63/64, weight 292.6 grams.

Time.	*Temp.*	*Barom*	*Pwdr.*	*Flight.*
A. M.	C.	mm.	c. c.	in.
10.20	23°.9	764.9	26	⅜ high
10.42	23°.8	"	25	Central
11.00	23°.8	"	25	"
11.18	23°.9	"	25	"
11.30	24°.	"	25	1½ high
11.40	24°.1	"	24	½ high
11.50	24°.2	"	23	Central
12.00	24°.3	"	23	¼ high
12.10	24°.3	"	23	" "
12.25	24°.4	"	22	Central
12.35	24°.4	"	22	⅛ low
12.45	24°.4	"	22	¼ "

The Records.

A full size sketch taken from a regular plate full of 17 inch long records, is shown in Fig. 13. The zero marks are seen as groups of three dots near the bottom of the print; and the bullet interruptions near the middle of the rectilinear tracings. The distance from the first dot to the first interruption shows how long the plate was falling before the ball cut the first sunbeam. Similarly for the others; hence the time of passage from beam to beam is accurately revealed if these distances can be precisely measured. This is easily done; for the interruptions are sharp, the tracings are strong and on transparent ground; they are, moreover, but 0.15 mm. broad, and 0.5 mm. apart from center to center, so that the three can be seen at one time in the field of the dividing-engine microscope.

TABLE II.

Measurement of Records of July 22, 1897.—Readings in Millimeters.

Record.	ZEROS.			BREAKS. *						Fall of Plate.
	1st.	2nd.	3rd.	1st.		2nd.		3rd.		h_1, h_2, h_3.
1	.610	.620	.620	117.197	117.975	139.751	140.662	164.422	165.447	116.978
	.609	.620	.620	117.201	117.976	139.754	140.660	164.419	165.445	139.587
										162.513
2	.696	.752	.775	117.545	110.187	133.981	134.596	151.656	152.320	117.141
	.699	.750	.774	117.549	110.196	133.982	134.590	151.651	152.320	133.587
										151.212
3	.656	.720	.730	97.731	98.573	121.221	122.188	147.426	148.541	97.446
	.657	.721	.732	97.731	98.571	121.222	122.189	147.416	148.544	120.974
										147.251

To measure the records the plate is held by many supporting blocks on the dividing-engine table to keep it straight, as it was in the plate holder when the records were taken. A small mirror on the engine table enables one to view the records by transmitted light. The table is moved so as to bring each of the zeros in succession centrally under the cross-hair of the microscope and readings taken. Similarly the edges of each interruption are located which give its center This corresponds to the time when the bullet was half way across the sunbeam even if the beam was not cut

* In the first column of Table II, headed "Breaks", are given the dividing-engine readings for the edges of the first break in the record. Taking the mean of these and subtracting the first zero reading gives h_1, the fall of the plate Likewise for the others.

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centrally. To show the accuracy of the dividing-engine readings a table is presented giving their actual value for several settings. See table II.

An inspection of the table shows that no difficulty is found in setting the cross-hair at the center of the zero dots. Naturally the greatest difficulty is found in locating the edges of the third inter-

TABLE II—Continued.

Measurement of Records of Sept. 9, 1897.

Record.	ZEROS.			BREAKS.						Fall of Plate.
	1st.	2nd.	3rd.	1st.		2nd		3rd.		h_1, h_2, h_3.
1	.671	.623	.634	414.294	415.920	450.689	452.410	489.456	491.040	414.574
	.670	.622	.635	414.285	415.923	450.683	452.412	489.462	491.040	450.925
										489.615
2	.592	.542	.527	384.535	386.435	427.009	429.003	472.166	474.257	384.894
	.591	.540	.527	384.536	386.435	427,010	429.002	472.166	474.257	427.465
										472.684
3	.491	.432	.432	366.285	357.920	393.083	394.793	432.158	433.770	356.612
	.487	.430	.430	366.280	357.920	393.085	394.790	432.158	433.770	393.507
										432.533

ruption; for the ball does not usually cut the third sunbeam so centrally as the others. Thus in record No. 3, page 28, the bottom edge of the third interruption could not be accurately fixed, hence the entire record, if not rejected, was marked with suspicion. Each of the readings here differs from the mean by .005 mm. If we regard this as the error in locating the *edge*, the error in locating the *center* of the interruption would be one-half as much, supposing the top margin to be located with perfect accuracy.

As the plate where the interruption occurs is moving about 2500 mm. a second, the time value of the above mentioned error in locating the break-center must be .0025/2500 of a second, or one millionth of a second. With solid oak balls the time of transit from the second to the third sunbeam is about one hundred millionths of a second longer than the time of transit from the first beam to the second; hence the above time error would be one per cent. of the quantity that must be measured to find the bullet's retardation. When the record edges can be more exactly located, as in the other records, the time *error* is less than one millionth of a second; and with hollow ash balls, the time *interval* which reveals the retardation is usually about 250 millionths of a second. A quantity which can,

therefore, be measured accurately to less than one half of one per cent; which means that the resistance (See note 2) can be determined with corresponding accuracy, when every precaution is taken.

It may be interesting to examine here the ideal and the practical accuracy of this method of measuring the projectile velocity. The time of transit from the first beam to the third is $14/560 = 1/40$ of a second, for a ball moving 560 feet a second, the sunbeams being 7 feet apart. Hence the instrument, which measures one millionth of a second, will, if the plate falls in vacuo without friction, measure this quantity accurately to 1/250 of one per cent; and if the sunbeams were placed four times as far apart, the velocity could be measured accurately to 1/1000 of one per cent. This is true, of course, for all balls, light or heavy. In practice the error made in measuring projectile velocities, owing to friction, is less than 1/20 of one per cent.

Calculations.

Having measured the distances h_1, h_2, h_3, through which the plate has fallen when the ball is midway across each beam, the times of falling, t_1, t_2, t_3, are found by the formula $t=(2h/g)^{\frac{1}{2}}$ for free fall in vacuo, which I shall show later to be legitimate for this instrument. See note 3. From the times t_1, t_2, t_3, we obtain the projectile velocities $v_1=\frac{7}{t_2-t_1}$, $v_2=\frac{7}{t_3-t_2}$, at the middle of the first and second seven foot stages, which values are very *approximately* (See Note 1.) true for such small portions of the bullet's path. From this we have v_1-v_2 for the change of velocity between the middles of the first and second stages. Dividing the change of velocity by the time in which it occurs, i. e. the time of passing from the middle of the first stage to the middle of the second, gives the retardation, which multiplied by the mass of the projectile gives the resistance of the air. This is but an application of the simple formula of mechanics

$$F=m\frac{v_1-v_2}{t_1-t^2}, \text{ or } F=m\frac{dv}{dt},$$

F being the force, m the mass, *i. e.* the force equals the mass times the acceleration.

Another method is obtained by writing the work done by the ball equal to its loss of energy, thus: $7F=\frac{1}{2}m(v_1^2-v_2^2)$*, whence $F=\frac{1}{14}m(v_1+v_2)(v_1-v_2)$, which is the formula I have usually employed and a convenient one for logarithmic computition. Of course, to

* In the limit this equation is $Fds=mvdv$, a familiar equation in mechanics.

find merely the resistance, it is not necessary to obtain either the t's or the v's; for F can be directly expressed as a function of the h's; but in order to plot the relation between the velocity and resistance it is necessary to know the velocities also.

A very good method of finding the retardation is to measure the projectile velocity at many intervals along its path, then construct a curve co-ordinating space and time, and find the equation of the curve, i. e. $s=f(t)$, whence $v=ds/dt$. From this we derive at once the retardation $dv/dt=f''(t)$, which multiplied by the bullet's mass gives the resistance. I did not employ this method for want of a sufficient number of light-beams to establish the space-time curve.

By using a seven decimal table of logarithms and arranging all the work concisely, it is possible with, ordinary skill, to accurately compute t_1, t_2, t_3,, v_1, v_2, v_3, and F, for one shot, in about twenty minutes to half an hour, and review the work; which is about the time required to measure the record on the dividing engine.

Results.

The accompanying table (See table III.) gives the essential data and all the values that must be computed for a four inch ball obtained from ten records. The values of Δt are particularly interesting as these small intervals of time must be measured accurately to one per cent in order to determine the resistance accurately to one per cent. See note 2. The small changes of velocity, Δv, must also be measured accurately to one per cent. The resistance values in the last column, also those following the table, are given without correction for temperature and barometric pressure; but these corrections, would not, if made for the mean temperature of the experiment, materially alter the curve of resistance plotted from the table and shown in Fig. 14; however, they will be attended to before the research is finished.

The curve is steeper than the nearest parabola of the family $R=a\,v^2$, and more closely coincides with a curve of the family $R=a\,v^3$; but more nearly still with a curve represented by the equation $R=a\,v^2+b\,v^3$, as manifested by Fig. 14, which is the exact graph of the equation $R=0.0008\,v^2+0.000000049\,v^3$. Thus the law of atmospheric resistance so earnestly maintained on analytical grounds by Col. Duchemin, early in this century, and controverted by nearly all later experimenters, seems to be corroborated by the measurements made in this research, as far as they go. It would be most interesting, therefore, to have records at higher and lower velocities.

TABLE III.

Resistance of 3'' 63/64 Sphere.

Mass of Ball	*Temp.*	*Barom.*	h_1	h_2	h_3	τ_1	τ_2	$\Delta\tau$	v_1	v_2	Δv	v.	R.
gm.	deg. C.	mm.	mm.	mm.	mm.	sec.	sec.	sec.	ft. sec.	ft. sec.	ft. sec.	ft. sec.	lbs.
230.2	24.6	761.1	208.533	270.325	340.851	0.028590	0.028872	0.000282	244.84	242.45	2.39	243.68	1.313
229.2	24.8	761.4	239.554	285.422	339.554	0.021662	0.021898	0.000236	323.14	319.66	3.48	321.40	2.482
200.0	24.2	760.1	271.725	314.901	361.947	0.018024	0.018281	0.000257	388.37	382.91	5.46	385.64	4.124
359.6	25.2	761.6	346.863	389.618	435.251	0.015914	0.016046	0.000132	439.86	436.25	3.62	438.05	5.582
226.7	24.1	762.2	301.296	336.740	374.704	0.014181	0.014390	0.000211	493.62	486.45	7.17	490.03	7.802
163.5	26.9	759.8	414.574	450.925	489.615	0.012484	0.012746	0.000262	560.718	549.192	11.526	554.95	10.243
226.7	26.7	760.2	256.457	282.905	311.143	0.011509	0.011700	0.000191	608.20	598.29	9.91	603.24	13.274
226.7	24.2	762.8	264.426	293.274	324.073	0.009166	0.009333	0.000167	763.69	750.03	13.66	706.86	21.796
224.6	24.6	761.5	172.468	187.467	203.458	0.007989	0.008173	0.000184	876.21	856.48	19.73	866.34	37.602
226.7	26.6	759.7	200.372	215.361	231.261	0.007428	0.007603	0.000175	942.38	920.69	21.69	931.53	44.862

h_1, h_2, h_3 = distances from zeros to record interruptions.

τ_1, τ_2 = times of transit between sunbeams.

v_1, v_2 = velocities midway between sunbeams.

$v = \frac{v_1 + v_2}{2}$, means velocity of projectile.

R = resistance of projectile in pounds weight.

In addition to the above tabulated values the following also have been obtained:

v=285.64 302.65 293.26 344.83 366.32 393.94
R= 1.790 2.286 2.182 2.925 3.473 4.405

v=426.00 461.92 531.33 562.82 582.35 628.09
R= 5.306 6.784 9.087 10.411 11.586 15.473

v=655.56 669.53 677.42 691.32 723.18
R= 18.164 19.998 20.419 21.085 23.896

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I hope later to use a vertical cast-iron gun and screen, of greater power and accuracy, to determine the resistance at all speeds from zero to 1300 feet a second, in order to discover how Duchemin's equation accords with the data of experiment throughout this range of velocities. If this equation proves true for all these speeds it will be a triumph for Duchemin's long neglected analysis and a remarkable instance of how mathematical theory sometimes anticipates the results of experiment. His law of resistance of inclined planes was published at the same time and corroborated by the elaborate experiments of Dr. Langley, fifty years later. Possibly his law of resistance of the sphere will be experimentally established too, notwithstanding that it has been negatived by the most accurate ballistic researches made during the past century.

Corrections.

The chief corrections to be made are those for temperature and barometric pressure. Since the viscosity of air is negligible, its resistance varies directly as its density, which varies about one per cent for a change of 3^0 C. in temperature, or 7.6 mm. in barometric pressure. Hence it is seen that these corrections are easily made if it is desired merely to keep the error below one per cent, as has been the aim in this investigation. The actual record of temperatures shows a variation of less than 3^0 from the mean; the pressures vary less than 7.6 mm. from the mean.

Sources and Limits of Error.

There are, from other sources, errors which it would be difficult to evaluate, which therefore, I have tried to minimize sufficiently to render them negligible. The most noteworthy of these are: (1) the rugosity and non-sphericity of the ball; (2) the blast behind it; (3) the resistance of the falling photographic plate; (4) the wobbling of the plate during its descent; (5) the imprecision of the record edges; (6) the refraction of the sunbeams.

(1) The error due to the roughness of the ball's surface is eliminated by wiping the ball and sponging the gun after each shot, and discarding each ball as soon as injured. The change of sphericity due to shrinkage or expansion is disposed of by callipering the ball from time to time, and rejecting it before its size or shape is seriously altered. As the best hollow balls seldom outlive a dozen shots covering, at most, two or three hours, and as they are finished in the lathe just before use, there is little danger from shrinkage.

A very slight change of mass also, occurs from moisture and powder; but a platform balance is kept at hand to test this. The variation seldom reaches ⅓ per cent. of the ball's mass.

(2) Care has been taken to ensure that the ball shall move through quiet air while flying from the first to the last sunbeam. The air of the room is still and the gases so checked by the eight screens before the gun that the smoke cannot reach beyond the sixth one for any charges thus far recorded, though for larger charges the smoke would doubtless extend further. Between the seventh and eighth screen and a foot to one side, a sheet of suspended newspaper is just perceptibly disturbed, while beyond the eighth screen at the first sun-beam the draft due to the passing bullet will blow out a candle held a foot from the trajectory. The latter effect is likewise observed at the third sun-beam; hence to test the equality of drafts at the first and third beams, I placed a row of equal gas-jets in each beam. When the ball passed, the same number of jets were blown out at each sunbeam, which seems to indicate that the conditions are the same at both places; i. e. that the draft at the sun-beams is due only to the passing ball, and that the gun-blast is therefore negligible.

(3) In order to diminish the resistance to the falling plate, it was originally intended to exhaust the camera of air by keeping it connected with one of the vacuum-cocks of the laboratory; but, on second thought, it seemed much simpler to use a heavy plate-holder, so as to nullify the effect of the air resistance. A rough calculation showed that, with its end rounded, the plate-holder, at its speed when the interruptions are made, meets a resistance of about 1/800 of a pound if falling in free air; so the plate holder was made of a wrought-iron bar, weighing, with plate and clamp, about 15 pounds. Thus the force of gravity urging this mass downwards is $800 \times 15 = 12000$, times greater than the air resistance to its fall in free space. Inside the box the diminution of the acceleration due to the air resistance may be something more than 1/12000 that of gravity; but it is evidently negligible, since only a resistance approaching one per cent of the weight need be considered in finding the retardation accurately to one per cent.

The friction of the plate-holder against the grooves which it brushes in its descent, was also proved inconsiderable by balancing it against a nearly equal weight so that it was free to fall plumb down. A small fraction of an ounce was sufficient weight to overcome the friction of descent; hence this also is negligible, since the total mass is 240 ounces. It can easily be demonstrated that, even if the

resistance to falling be as great as 1/1000 of the weight of the falling mass, the error introduced into the calculation of the projectile velocity and resistance will be but a small fraction of one per cent, whether the resistance be regarded as varying uniformly or in the most irregular manner possible. See note 3.

(4) The wobbling of the plate causes a more serious error and is of two kinds; (*a*) wobbling in the plane of the plate; (*b*) to and from the lens.

(*a*) In order to prevent excessive wobbling in the plane of the plate, or plane of the grooves, the latter were made very truly parallel and equidistant. After the grooves were planed as accurately as I could have the work done in a Washington machine-shop, an expansible drift, made in the laboratory, was run back and forth through the grooves until they were cut as truly equi-distant as could be determined by a sleeve calliper — an instrument which can readily detect an error less than 1/1000 of an inch in linear magnitude.

Now, since the plate-holder touches only at its top and bottom against the grooves of the camera, it is easy to see what effect wobbling has on the relative motion of plate and image. For suppose the lower left corner of the plate-holder to be a fixed point of rotation and the top of the plate is made to play back and forth in the plane of the grooves. Evidently every point on the plate will have some upward and downward motion, this being a maximum along the right side of the plate. The distance between the top and bottom bearings of the plate-holder is 24 inches, the distance from the left bearing to the right side of the plate is 4 inches; hence if the plate at its top vibrates 1/1000 of an inch about the lower left bearing as a fixed center, every point of the right edge of the plate will rise and fall 4/24 . 1/1000 of an inch or 1/6000. This is the maximum vertical displacement that can arise from such vibration, while the minimum, i. e. that along the left edge of the plate, is about 1/18000 of an inch.

This estimate shows that as the plate wobbles in its plane while descending, it may happen that its vibrations are in different phase at successive interruptions, and thus introduce an error whose greatest possible value is just negligible, though not easily so. Of course the phases could not always be the most unfavorable, nor could the fixed axis about which the plate is at any instant vibrating, always be the most unfavorable. Hence, on the average, these displacement errors would be much less than 1/6000 of an inch; so that, even if they were always of the same sign, they might be disregarded.

But they are as likely to be positive as negative, and so tend to neutralize each other when an average of many records is taken, which is an additional reason why they may be neglected. It may be added that whenever the axis of vibration of the freely falling plate happens to be near the path of the image on the plate, the relative displacement of plate and image is practically nothing.

(*b*) The analogous displacement error due to the wobbling to and from the lens is seen, by the same reasoning, to be about 1/16 of the error obtained above, since the face of the plate is about ¼ of an inch from the plane of the grooves.

Another slight error due to this motion is caused by the change of size of the sun-beam images as the plate moves to and from the lens. For each image is approximately the vertex of a right pyramid of light whose base is on the lens and is ⅛ of an inch square, while the length of the pyramid is about 2¼ inches. As the ratio of these two dimensions is 1/18 it is evident that a displacement of the plate 1/1000 of an inch, to or from the lens, enlarges or diminishes the images by 1/18000 of an inch, which therefore, is the maximum displacement error due to such wobbling. Hence the mean error, which is much less than this, may easily be disregarded.

(5) The sharpness of the margins of the record interruptions depends upon the suddenness of the sun-beam eclipses and the size of the images. If the beams are 1/100 of an inch thick and ⅛ of an inch in vertical width, a "straight" ball moving 500 feet a second will cut each beam in $1/100 \times 1/500 \times 1/12 = 1/600{,}000$, of a second, during which interval the plate moves 1/6000 of an inch, since its speed is about 100 inches a second when the interruption occurs. If the ball cuts the beam obliquely, more time is required to extinguish and restore the light, the periods of extinction and restoration being exactly equal for a bullet which is symmetrical fore and aft. If the ball goes centrally through one sun-beam but slightly to one side of the next, practically no error is introduced, one record interruption being merely made shorter than the other, while its center is the same as it would be for a perfectly straight shot. But if lhe ball goes very much aside at either beam, as shown by the record margins, or by the target paper, the entire record should be rejected because of the imprecision of its edges. So long, however, as the margins can be clearly located, the record is used, even if the ball should strike one or more beams non-centrally to the amount of ⅛ of an inch or so.

Of course the interruption margins would not be perfectly sharp, even if the light were extinguished in an infinitesimal time,

unless the images were infinitely narrow up and down. For, as the square image reaches any point on the plate, first the top of the image actinizes the point, then the middle of the image, then the bottom, the first rays which reach the point being much the most effective. Thus it appears that if the light were interrupted in no time, the end of the tracing would still shade off, having a margin about as broad as the decomposing image. But since the margins at the beginning and the end of the interruption are close together and alike, it is easy to set the cross-hair similarly at the edge of each margin, and thus to find the desired center of the interruption with great exactness. This has already been shown by the dividing-engine records.

(6) It may be well to consider a possible sixth source of error. It seems that the compressed air before the ball must refract each beam of light and throw it off the reflector slit before it is cut by the ball, while the rarefied air behind the ball does not produce a compensating effect in restoring the light as the ball emerges from the beam. This might be a cause of considerable error if the beam were simply a fine ribbon of light; but as it is a cone whose rays fall on both sides of the slit, the effect of such refraction is merely to bring new rays upon the slit. Hence the light can be extinguished at the slit only by the ball actually cutting the sun-beam, and the instant of this cutting can be hastened only by the rays bending in towards the ball within the small region of compressed air before it; in other words, by the side displacement of the beam along the bullet's path. Now as the impactual pressure of the air against the entire front of the ball is at most some 40 lbs. in these experiments, the pressure of the densest part must be much less than one atmosphere above the reading of the barometer; and as the index of refraction for air is but 1.002922, under normal conditions, it is apparent, and can be analytically demonstrated, that, even though the displacement of the light may be considerable at the reflector slits, its displacement just in front of the ball, which is the critical place, is much less than the width of a slit, and therefore negligible.

I have a simple plan for measuring the deviations of the beam during the entire approach and recession of the ball by means of the gravity camera, and hope to apply it when I can find the necessary time.

These are the main sources of inaccuracy, all of which combined should not cause an error of one per cent in the determination of

tne resistance, if the apparatus is in perfect condition and due care is taken throughout the experiment.

Conclusion.

I have now practically accomplished for the sphere what was originally undertaken for surfaces of revolution in general of convenient form. The method devised has proved satisfactory; but whether it can be successfully applied to spheroids, cylinders, cones, ogival shaped projectiles, etc., is questionable. Unless such shapes are made to preserve their allignment during flight the sun-beam interruptions will not occur under identical conditions, and hence they cannot serve to determine with accuracy either the bullet's velocity or its retardation. Whether such allignment can in every case be preserved by weighting the end of the projectile, or by causing it to spin from a rifled gun; or whether the resistance can be truly determined if the projectile is guided along a rod or wire, remains to be ascertained by further experiment.

MISCELLANEOUS NOTES.

NOTE I.

Average Velocity and True Velocity.

On page 30 the assumption is made that a bullet's velocity at the middle of a short element of its path is very approximately equal to its average velocity along the element. It is required to determine the exact ratio of these velocities for any law of atmospheric resistance. In other words it is required to evaluate the ratio $v \div \frac{2e}{t_2 - t_1}$, in which $2e$ is the length of an element of the bullet's path, v its velocity at the middle of the element and $t_2 - t_1$ its time of traversing the element.

Let $t = f(s)$; t being the time, s the space described, since the retardation began. Then the times at the beginning and end of the element of path may be written:

$$t_1 = f(s-e) = f(s) - ef'(s) + \frac{e^2}{2!}f''(s) - \frac{e^3}{3!}f'''(s) + \ldots$$

$$t_2 = f(s+e) = f(s) + ef'(s) + \frac{e^2}{2!}f''(s) + \frac{e^3}{3!}f'''(s) + \ldots$$

$$\therefore t_2 - t_1 = 2e\left[f'(s) + \frac{e^2}{3!}f'''(s) + \frac{e^4}{5!}f^{v}(s) + \ldots\right],$$

$$\text{and } v \div \frac{2e}{t_2 - t_1} = v\left[f'(s) + \frac{e^2}{3!}f'''(s) + \frac{e^4}{5!}f^{v}(s) + \ldots\right]. \quad (1)$$

Now, whatever the law of atmospheric resistance, $t = f(s)$ gives $v f'(s) = 1$; hence, writing r for $v \div \frac{2e}{t_2 - t_1}$, equation (1) becomes:

$$r = 1 + v\left[\frac{e^2}{3!}f'''(s) + \frac{e^4}{5!}f^{v}(s) + \ldots\right]. \quad (2)$$

If, therefore, the law of atmospheric resistance is such that the second term in the value of r is zero, or negligible, the average velocity between the points may be taken as the mean velocity along the element of the path.

Discussion of Equation. (2)

Functions for which $r=1$.—The only functions of the form $t=f(s)$, whose third and higher derivatives are zero, are of the form:

$$t=a, \text{ or } t=s/\infty + a, \qquad (\alpha)$$
$$t=as + b, \qquad (\beta)$$
$$t=as^2 + bs + c. \qquad (\gamma)$$

(α) gives an infinite velocity and zero retardation, i. e. $ds/dt=\infty$; $d^2s/dt^2 = o$.

(β) gives a constant velocity and zero retardation.

(γ) gives a retardation which increases directly as the cube of the velocity, i. e. $d^2s/dt^2 = -2av^3$.

Thus for each of the laws of resistance expressed by (α), (β), (γ), the average projectile velocity along an element of path exactly equals the velocity at the middle of the element.

Now it so happens that equation (γ) nearly represents the velocity resistance curve obtained by experiment. If it represented the curve perfectly the assumption that the average velocity between two points equals the actual velocity midway between them would be, not only approximately but rigorously, correct for horizontal flight in homogeneous, still air.

Functions for which $r\neq 1$.—There is an infinite number of functions of the form, $t=f(s)$, whose third and higher derivatives are not zero nor such as to make the second term in the value of r vanish; but any of the functions proposed by the various writers to express the law of atmospheric resistance would give a value of r very nearly equal to unity for a value of $2e$ so small as 7 feet. Of all these, Duchemin's equation $R=av^2 + bv^3$, most nearly expresses the law of resistance manifested by this research. Applying to this the general formula (2), it is easily seen that r (the ratio of the average and mid velocity for an element of the bullet's path) differs from unity by very much less than one per cent, for the conditions of the experiment.

Thus solving the equation

$$\frac{d^2s}{dt^2} = mv^2 + nv^3,$$

and writing c for the constant of integration, we derive

$$\frac{v}{m + nv} = e^{m(s-c)};$$

whence $$\frac{d^3t}{ds^3} = \frac{m}{e^{m(s-c)}}, \text{ and } \frac{d^5t}{ds^5} = \frac{m^3}{e^{m(s-c)}}.$$

In practice we may have $m=0.03$, $n = 0.000002$, and midway

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between two sunbeams, $r' = 500$. Using these values to determine the above derivatives, then substituting in equation (2), we obtain $r = 1.00018+$. This proves that the mid velocity is about 1/50 per cent greater than the average velocity between the sun beams, for the light wooden balls employed.

NOTE II.

Relation between Bullet's Retardation and $\Delta\tau$.

Promblem (1).—To prove that the bullet's retardation is proportional to the difference between its times of transit across the first and second sunbeam stages.

Let the average velocity across the first 7 foot stage be v_1, across the second stage v_2; let the corresponding times of transit be τ_1 and τ_2. Then as the ball passes from the middle of the first stage to the middle of the second, the change of velocity is very approximately $v_1 - v_2$, which occurs during the time $\frac{1}{2}(\tau_1 + \tau_2)$. Hence the retardation equals $(v_1 - v_2) \div \frac{1}{2}(\tau_1 + \tau_2)$. Now, writing $\tau_2 = \tau_1 + \Delta\tau$, we have

$$v_1 - v_2 = \frac{7}{\tau_1} - \frac{7}{\tau_1 + \Delta\tau} = \frac{7\,\Delta\tau}{\tau_1^{\,2}},$$

very approximately. Hence the retardation is

$$\frac{7\,\Delta\tau}{\tau_1^{\,2}} \div \frac{\tau_1 + \tau_2}{2} = \frac{7\,\Delta\tau}{\tau_1^{\,3}},$$

very approximately. This shows that the retardation is proportional to $\Delta\tau$, i. e. proportional to the difference between the times of transit across the sunbeam-stages.

Application of Problem (1).—To determine, therefore, the resistance accurately to one per cent it is clear that the chronograph must measure $\Delta\tau$ accurately to one per cent, or less. Now the value of $\Delta\tau$ is usually in the neighborhood of 200 millionths of a second for wooden balls and speeds of a few hundred feet per second; hence the chronograph should measure this interval of time accurately to one per cent, or less; that is the greatest time error admissible is two millionths of a second. How nearly the chronograph meets this requirement may be judged from the velocity-resistance diagram, Fig. 14, where the ordinates represent resistances, and the uniformity of their values shows approximately the accuracy with which the chronograph measures $\Delta\tau$. As there are other small errors in finding the resistance besides those of the chronograph, for example errors due to the varying atmospheric density, the varying mass,

shape and smoothness of the ball, etc.; it is obvious that the measurement of $\Delta\tau$ is even more accurate than shown by the uniformity of the ordinates in the diagram.

NOTE III.

Plate Friction and Time Error.

Problem (1).—To prove that the error due to neglect of friction, in computing the time of fall of the photographic plate, is negligible.

We may compute the time of free fall and the time of fall when the friction is constant and equal to the maximum that occurs in practice. If their difference is considerably less than one per cent of the whole time of fall the error is negligible, since it will cause an error of less than one per cent in determining the velocity of the projectile.

Let g be the acceleration of gravity, ng the maximum retardation of the plate, and h its fall from rest. Then

$$t = \left(\frac{2h}{g}\right)^{\frac{1}{2}}, \text{ is the time of fall without friction;}$$

$$t' = \left(\frac{2h}{g-ng}\right)^{\frac{1}{2}}, \text{ is the time of fall with friction;}$$

Hence $$\frac{t'-t}{t} = \left(\frac{1}{1-n}\right)^{\frac{1}{2}} - 1 = (1-n)^{-\frac{1}{2}} - 1 = n/2 + \ldots,$$

is the proportional error due to friction. In practice $n < 1/1000$ and may be very much less; hence, neglecting higher powers of n, we have

$$\frac{t'-t}{t} < \frac{n}{2} = \frac{1}{2000} = \frac{1}{20} \text{ per cent.}$$

This proves that the error in question is easily negligible.

Problem (2).—To find the error due to the friction of the plate-holder while falling from the first break to the second of the record.

Let d be the distance from the first to the second break, τ the time of traversing d without friction, τ' the same with friction, ng the average frictional retardation, v the average velocity without friction Then the average velocity with friction is $v - ng\tau/2$, and the absolute error due to neglect of friction is

$$\tau' - \tau = \frac{d}{v - \frac{1}{2}ng\tau} - \frac{d}{v} = \frac{ng\tau d}{2v^2} = \frac{ng\tau^2}{2v},$$

approximately; while the proportional error is $(\tau' - \tau)/\tau = ng\tau/2v$. In practice we may have $n < 1/1000$, $v = 12$ ft. sec., $\tau = 1/70$ sec.

Hence

$$\tau' - \tau = \frac{ng\tau^2}{2v} < \frac{1}{1000} \times 32 \times \frac{1}{(70)^2} \times \frac{1}{24} = \frac{1}{3{,}600{,}000} \text{ sec.},$$

is the absolute error; and

$$\frac{\tau' - \tau}{\tau} < \frac{70}{3{,}600{,}000} = \frac{1}{500} \text{ per cent},$$

is the proportional error.

Application of Problems (1) and (2).—Problems (1) and (2) may be used to evaluate the error in computing the difference between the times of fall of the plate from the first to the second break, and from the second to the third. The evaluation of this error is very important because the bullets retardation, and therefore its resistance, is directly as the difference of its times of transit along the first and second light beam stages (Note 2). This differential error is manifestly negligible; for since the greatest possible error for either stage, due to neglect of friction, is 1/3,600,000 of a second in the average case assumed above, the error in finding the *difference* between the times for each stage cannot be greater; and must, on the average, be much less, since the friction of the falling plate is nearly constant for so small an interval. It would be more nearly correct to say that the proportional error in computing the difference of the two times, is the same as for the times themselves. In practice the difference $\Delta\tau$, in the times of transit of the projectile across the light-beam stages, is 200 millionths of a second; and, if the proportional error for each transit is 1/50,000, as above, the absolute error due to neglect of friction during these small intervals, cannot exceed $\frac{1}{50{,}000} \cdot \frac{200}{1{,}000{,}000} = \frac{1}{250{,}000{,}000}$ of a second. Of course the main friction error in finding the value of $\Delta\tau$ is due, not to the retardation of the plate while falling from break to break, but to the retardation while falling through the long distance preceding the breaks. This error was shown in problem (1), to be less than 1/20 per cent of 200 millionths of a second, or 1/10,000,000 of a second.

It has seemed advisable to evaluate these errors somewhat carefully because several critics of the chronograph have thought that they might be great enough to be detrimental, particularly the last one considered which has proved to be so extremely small.

When the plate holder does not touch the side grooves in its

descent, a more exact value of the friction error is given by the equation of a body falling from rest in air; thus:

$$\frac{d^2 s}{d t^2} = g - \mathrm{k}\, v^2 .$$

Solving for t in terms of s, we obtain

$$e^{mt} + e^{-mt} = 2\, e^{ks}, \text{ in which } m = (g\,k)^{\frac{1}{2}}.$$

$$\text{Hence } 2\left(1 + \frac{m^2 t^2}{2\,!} + \frac{m^4 t^4}{4\,!} + \ldots\right) = 2\left(1 + k\,s + \frac{k^2 s^2}{2\,!} + \frac{k^3 s^3}{3\,!} + \ldots\right)$$

Neglecting powers higher than the square, since m and k are small; and solving for t we have

$$t = \left[\frac{2\,s}{g}\left(1 + \frac{k\,s}{2\,!}\right)\right]^{\frac{1}{2}}.$$

But for free fall from rest in vacuo

$$t' = \left(\frac{2\,s}{g}\right)^{\frac{1}{2}},$$

t' being the time. Therefore

$$\frac{t}{t'} = \left(1 + \frac{k\,s}{2}\right)^{\frac{1}{2}}.$$

In f. p. s. units k is about 0.0001 in this research; and s, where the breaks occur, is about one foot; thus making

$$\frac{t}{t'} = (1.00005)^{\frac{1}{2}} = 1.000025, \text{ very nearly.}$$

Thus both methods of approximation justify the assumption that the friction error is negligible in determining both the velocity and the retardation of the projectile.

NOTE IV.

Measurement of the Projectile Velocity Inside the Gun.

The methods described in the preceding pages for measuring projectile velocities outside the gun may, with slight modification, be used also to measure velocities inside the gun. To apply the electro-chemical method it will suffice to use a pole fixed to the projectile and protruding from the muzzle of the gun, or any other practicable interrupter hitherto employed inside the gun. To apply the photographic method a beam of light may be used running from

the projectile to the camera in such a way as to be instantaneously and momentarily extinguished, or restored, for several positions of the bullet along the bore.

Fig. 15, shows a device for photographically recording the bullet's arrival at successive stages along the bore of a rifled gun whose axis remains fixed during the first part of the recoil. A disk on the front of the projectile has several illuminated spots near its outer edge each of which throws light on a mirror M, before the gun's muzzle, the mirror being so inclined as to direct into the camera P, only rays incident parallel with the gun's axis. As the disk rotates, each luminous spot in turn comes suddenly into the favorable posi-

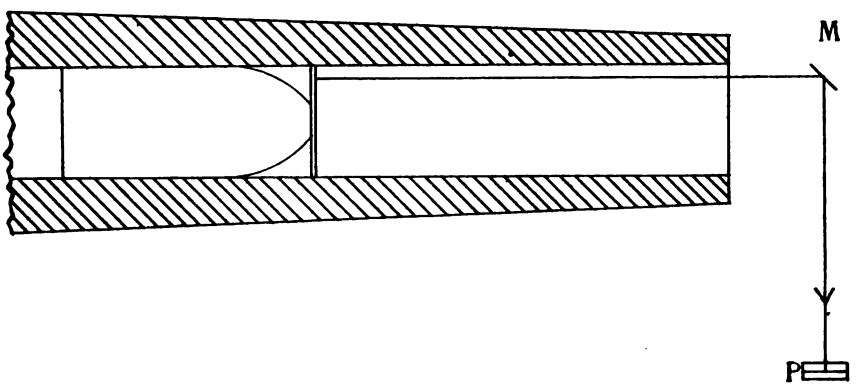

FIG. 15.—Plan for Measuring Projectile Velocities Inside the Gun.

tion and momentarily illuminates the camera hole; thus recording on the falling plate its arrival at known points along the gun-bore. The photographic tracing must accordingly consist of a zero-mark at the bottom and a number of dashes, one above the other, whose positions reveal the bullet's velocity, from which also the acceleration can be calculated if so desired.

The illumination of the disk may be effected either by throwing light into the gun from outside, or by generating it behind the disk, say by means of burning magnesium. It might also be feasible to locate the luminous spots on the projectile itself, thus dispensing with the disk.

Evidently there will be, from each self luminous spot, a beam continuously incident on the mirror, and describing within the gun a conic spiral surface as the disk screws forward, while the corresponding reflected beam will describe a similar surface outside the

gun. All these exterior surfaces will have one element in common, viz. the straight line joining the mirror and camera hole; hence as the whirling beams chase each other, like so many search lights, each in turn will sweep across the camera hole and should give a very sharp record of the projectile's instantaneous position.

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